

# Three-Channel Multiple Description Image Coding Based on Special Lattice Vector Quantization

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## Abstract

*In this paper, we use three-dimension lattice vector Z3 to design an index assignment for three-channel multiple description coding. In the labeling procedure, the side quantizers are based on the sublattice, but the central quantizer is not only based on the lattice but also combines some spheres to get a better tradeoff between the central distortion and side distortion. Then this scheme is used on wavelet image coding. The experiment results on gauss source and standard test image both show that this scheme is simple and effective.*

## 1. Introduction

Multiple description coding (MDC) places different encoded versions of a given block of source samples (it is called description) into several packets. Each description can provide a degraded version of the source independently, while more descriptions can reconstruct a better version of the source. Vaishampayan in [1] introduces the earliest practical multiple description scheme multiple description scalar quantizer (MDSQ). Just as single description vector quantizer can be constructed to outperform scalar quantizer, multiple description lattice vector quantization (MDLVQ) is proposed by Servetto, Vaishampayan, and Sloane in [2] and [3] to outperform MDSQ. An optimal design of MDLVQ can be found in [4].

Network congestion and delay sensibility pose great challenges for multimedia communication system design. Therefore, there is demand for coding approaches combining high compression efficiency and robustness. Multiple description (MD) coding creates such representations. The classical scheme often involves two descriptions. In [1], Symmetric MD lattice

vector quantization (LVQ) was introduced by Servetto, Vaishampayan and Sloane (known as the SVS technique) for two balanced channels. However, more than three descriptions are in demand because of an emerging application: robust transmission of compressed data over IP networks. Packets transmitted over IP networks are limited in size, and most practical applications would need more than two packets for transmission [2]. An extension of the SVS for more than two descriptions can be found in paper [3]. But the index assignment mapping is not adequately addressed. Three-channel multiple description based on the two-dimension lattice A2 is shown in [4]. But it is only for theoretical analysis and it gets too much redundancy in side channel for a better tradeoff. An optimal n-channel index assignment is given in [5], which gives excellent theory analysis and asymptotical results. However, the scheme is complex and the side channel distortion is far apart from the central channel distortion.

In this paper, we propose an index assignment based on three-dimension lattice vector Z3 and apply it on wavelet image coding. With higher dimension, we can get better tradeoff because each lattice has more neighbors. We also proposed a scheme that combines the Voronoi cell of Z3 lattice and the sphere to quantize the central channel for much better tradeoff. The experiment results show that the scheme can meet the requirement of the side channel without much compromise of central channel.

## 2. Three-channel multiple description image coding scheme

In this section, we first describe our coding scheme for images, and then we describe our index assignment and tradeoff adjusting in detail.

Figure 1 is the coding diagram based on wavelet image. After  $n$ -level wavelet transform, the image is transformed into  $3n+1$  subbands. Then the LL subband is encoded solely with LVQ scheme because of its importance. As for the other subbands, we deal with them according to their levels. That can be seen from Fig. 1. Each level is encoded with LVQ of different quantization step. By doing this, each triplet can be selected from the group (HL, LH, HH) easily. Most important of all, the wavelet coefficients in each level are close to each other, which is beneficial to the entropy coding subsequently. When receiving one description, we use the sublattice point to reconstruct the image. When receiving two channels, the midpoint of the two received sublattice points is the reconstructed value. Therefore, it is very simple.

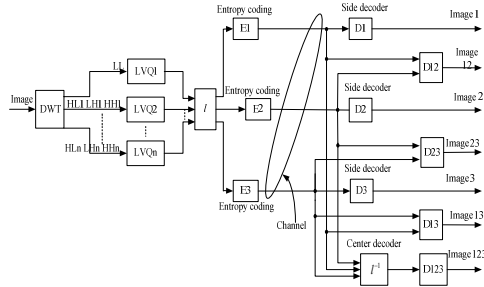


Fig. 1 Coding Diagram

## 2.1. Guiding Principle for three-channel index assignment

Like the SVS problem, our main scheme can be reduced to the problem of assigning three-tuple labels to points of a vector quantizer codebook. Therefore, the principle of index assignment is the most important. We selected the reuse index number as 27, which assure the sublattice is clean [6].

Since lattices are infinite arrays of points, in order to label a finite number of points, we construct a shift-invariant labeling function as that in [1]. The discrete Voronoi cell  $V_0(0)$  denotes the region of the sublattice around origin. Then we only need to label the finite lattice points in  $V_0(0)$ , the assignment can be extended to the entire lattice using shift property.

Assume that the fine lattice point  $\lambda$  the discrete Voronoi cell  $V_0(0)$  is mapped to a 3-tuple points  $(\lambda'_1, \lambda'_2, \lambda'_3)$  as illustrated in Fig. 2.

One can show that the side channel distortion mainly depends on the formula (1):

$$3D_{s1} = \|\lambda - \lambda'_1\|^2 + \|\lambda - \lambda'_2\|^2 + \|\lambda - \lambda'_3\|^2 \quad (1)$$

$D_{s1}$  denotes the side channel distortion. Based on the parallelogram and some induction, we can obtain

$$3D_{s1} = 1/3(\|\lambda - \lambda'_1\|^2 + \|\lambda - \lambda'_2\|^2 + \|\lambda - \lambda'_3\|^2) + 3\|\lambda - 1/3(\lambda'_1 + \lambda'_2 + \lambda'_3)\|^2 \quad (2)$$

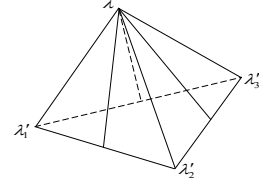


Fig. 2 A lattice point and its label  $(\lambda'_1, \lambda'_2, \lambda'_3)$

To minimize the overall side channel distortion  $D_{s1}$ , all the term in formula (2) should be minimized. Under a high-resolution assumption, the first term is dominant, from which we conclude that in order to minimize (2) we must use the three-tuple with the shortest edge (for details see reference paper [5]).

## 2.2. Index assignment

Then our principle of index assignment is the three edges of the three-tuple points  $(\lambda'_1, \lambda'_2, \lambda'_3)$  should be as short as possible while the centroid of three-tuple and the lattice point  $\lambda$  is as close as possible. In fact, the index assignment can meet the principle simultaneously, which can be seen from the index assignment table 1.

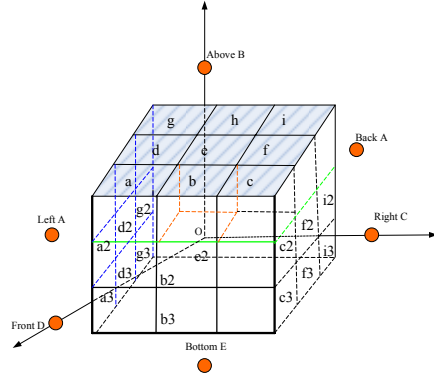


Fig. 3 A portion of the Z3 lattice

In Fig. 3, we show the lattice and sublattices. In fact, each large point is a large cube or a sublattice. And the small cube which represents the fine lattice is only showed in the origin sublattice  $V_0(0)$ . Lower case letters denote the sublattice point and upper case letters denote the lattice point. Using the index assignment principle above, we get the table 1 for our index assignment. In the table 1, each lattice point is mapped to a three-tuple sublattice point.

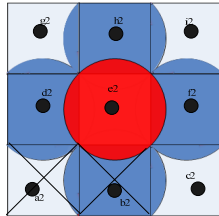
**Table 1 Index assignment**

|    |     |    |     |
|----|-----|----|-----|
| a  | ABD | f2 | OOC |
| b  | OBd | g2 | OAF |
| c  | BCD | h2 | OOE |
| d  | OAB | i2 | OCF |
| e  | OOB | a3 | ADE |
| f  | OBc | b3 | ODE |
| g  | ABF | c3 | CDE |
| h  | OBf | d3 | OAE |
| i  | BCF | e3 | OOE |
| a2 | OAd | f3 | OCE |
| b2 | OOD | g3 | AEF |
| c2 | OCd | h3 | OEF |
| d2 | OOA | i3 | CEF |
| e2 | OOO |    |     |

### 2.3. Adjusting the trade-off

Using the scheme above, we can see the performance of side description and central description are far apart, which likes SVS scheme. In order to get a better tradeoff between central and side distortions we proposed a scheme that combines the Voronoi cell of Z3 lattice and the sphere to quantize the central channel. Fig. 4 is a section figure of our combined scheme, which is cut from Fig. 3. Then the quantizer principle becomes:

Input(x)  
 If x belong to the sphere e2 which cover the whole cube e2,  
 Then it is quantized to e2.  
 Else if x belong to the sphere e or (b2, d2, f2, h2, e3),  
 Then it is quantized to e or (b2, d2, f2, h2, e3).  
 Else  
 The same with the Z3 quantizer  
 End



**Fig. 4 A section figure of our scheme**

Something to notice: the e2 is a sphere, but the b2 is not a sphere, e2 and another sublattice of Z3. So slice the b2 we recalculate its centroid. The centroid of a2 is recalculated too. We can extend the principle to the whole  $V_0(0)$  and the whole place.

Using this combined quantizer has one important advantage: It can change the coefficients probability of the side channel. Therefore, the entropy of side channel is smaller accordingly. Taking Fig. 4 for an example, more points will be quantized to the e2 and fewer point will be quantize to a2, so the entropy will be small, it will be easy to code this signal. Meanwhile, the central channel will not compromise much performance because the sphere Voronoi region has the smallest granular distortion. By tuning the radius of the sphere, we can tune the tradeoff between the central distortion and side distortion.

### 3. Numerical simulation and analysis

In the following simulation, the ideal MDC network is used, where we consider a description is either intact or completely lost. We compare our three-channel description scheme against the two-channel SVS scheme. The sources include the gauss signal and standard images.

Assuming channel loss probabilities are independent and the same for all descriptions, denoted by  $p$ . Then the expected distortion for three-channel system is given by:

$$D_3 = (1-p)^3 D_c + 3(1-p)^2 p D_{s2} + 3(1-p)p^2 D_{s1} + p^3 E[\|X\|^2] \quad (3)$$

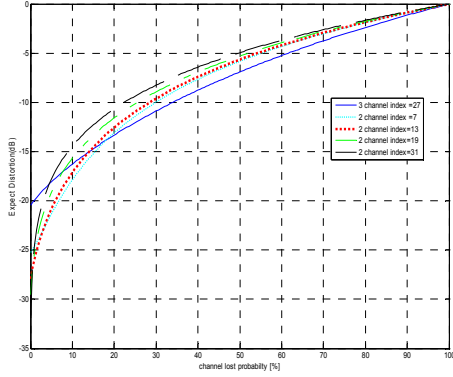
Where  $D_c$  is the central distortion when three channels work,  $D_{s1}$  is the average side distortion when only one channel works,  $D_{s2}$  is the average side distortion when two channels work,  $E[\|X\|^2]$  is the variance of the source. We give the expected distortion for two-channel multiple description too:

$$D_2 = (1-p)^2 D_c' + 2(1-p)p D_{s1}' + p^2 E[\|X\|^2] \quad (4)$$

#### 3.1. Gaussian Source

We use the unit-variance Gaussian source signal and let the total target entropy be 6 bits/dimension. Therefore, each channel of SVS scheme is 3 bits/dimension and our three-channel scheme is 2 bits/dimension. Then we sweep the packet loss probability in the range  $[0, 1]$ . Different from [4], we do not use the best index number for certain packet loss probability because we do not know the probability before we transfer the signal. Therefore, the experiment results maybe different from the paper [4]. Figure 5 shows the expected distortion of our scheme and the four different index assignment of SVS

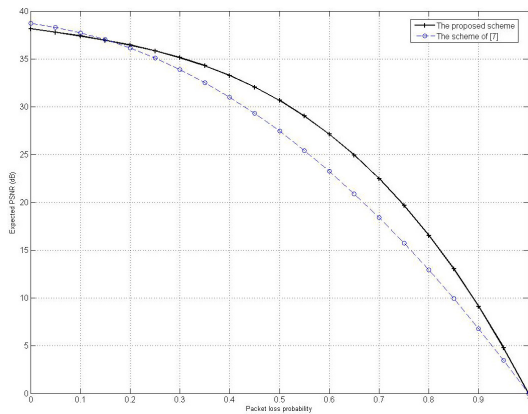
schemes for comparison. Our scheme is better than SVS scheme when the reuse index number is 31 at 5%. Moreover, our scheme is better than all reuse index number of SVS scheme when packet loss probability is larger than 18%.



**Fig. 5 Distortion functions of packet-loss probability for different index assignment**

### 3.2. Standard Image

We code the image Lena ( $512 \times 512$ ) with our scheme and compare the results with the paper [7] both at total bit rate of 1 bpp. It is assumed that at least one description is received and the conditional expected PSNR is selected as the guideline. Both the scheme in [7] and ours use the same Daubechies wavelet base. From the Fig. 6, we can see that when packet lost probability is larger than 15%, our scheme outperforms the scheme in [7].



**Fig. 6 Conditional expected PSNR value of our scheme and the scheme of paper [7]**

## 4. Conclusion

In this paper, an index assignment for three-channel multiple description based on the Z3 lattice is proposed.

By combining some spheres to quantize the lattice point, the tradeoff between central channel and side channel is adjusted. We use this scheme on the Gauss signal and wavelet image. The experiment results show that this scheme is a reasonable solution for MDLVQ and can be used on image coding.

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